Electrodynamics Ph.D. Qualifying Exam 2015/09/04 09:00–12:00

5 problems, total 100%

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1. (a) (10%)With Green's theorem, derive the equation

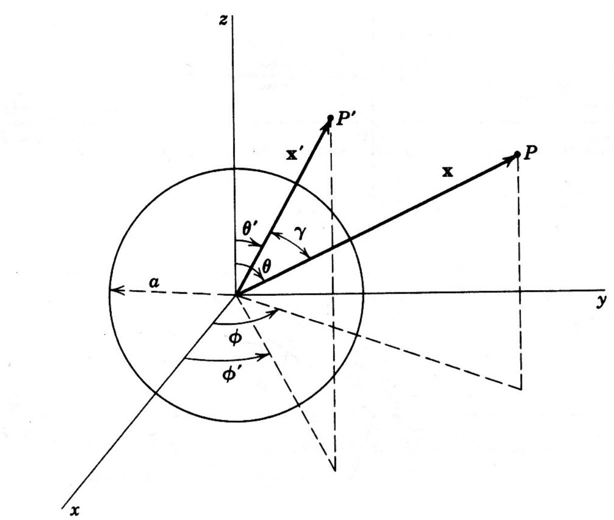
$$\Phi(\mathbf{x}) = \frac{1}{4\pi\epsilon_0}\int_V \rho(\mathbf{x}')G(\mathbf{x},\mathbf{x}')d^3x' + \frac{1}{4\pi}\oint_S \left[G(\mathbf{x},\mathbf{x}')\frac{\partial\Phi}{\partial n'} - \Phi(\mathbf{x}')\frac{\partial G(\mathbf{x},\mathbf{x}')}{\partial n'})\right] da',$$

where $\Phi(\mathbf{x})$ and $\rho(\mathbf{x})$ are the electric potential and charge density, respectively, at the point $\mathbf{x}$.

(b) (10%)Derive the equation of the potential for *Neumann boundary conditions*.

$$\Phi(\mathbf{x}) = \langle\Phi\rangle_S + \frac{1}{4\pi\epsilon_0}\int_V \rho(\mathbf{x}')G_N(\mathbf{x},\mathbf{x}')d^3x' + \frac{1}{4\pi}\oint_S \frac{\partial\Phi}{\partial n'}G_N da'$$

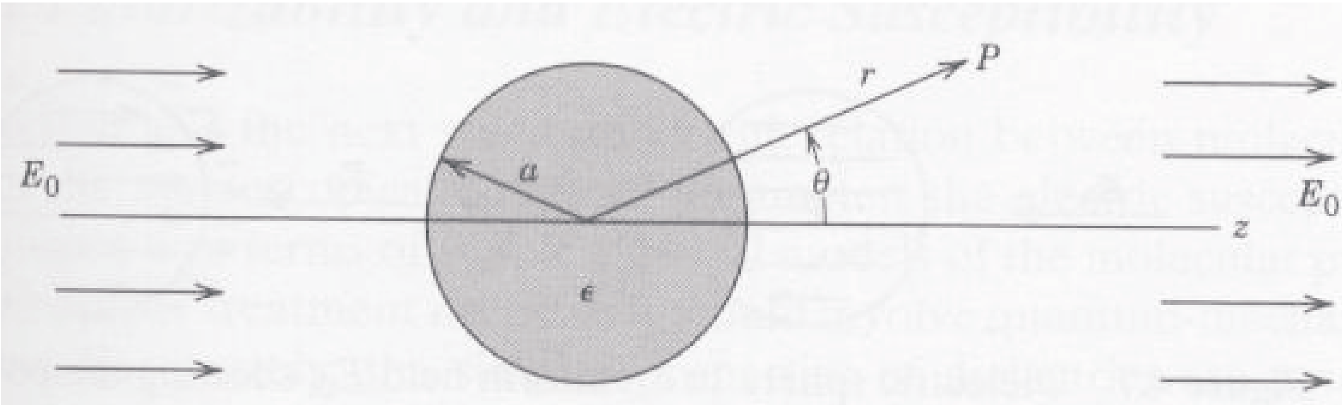
2. (20%)Consider the coordinates and the sphere shown in the figure. The potential on the boundary $r = a$ is $\Phi(a, \theta', \phi')$.



Show the potential at the *outside* point P is

$$\Phi(\mathbf{x}) = \frac{1}{4\pi}\int \Phi(a,\theta',\phi')\frac{a(x^2-a^2)}{(x^2+a^2-2ax\cos\gamma)^{3/2}}d\Omega'$$

3. (25%) Consider a dielectric sphere of radius a with dielectric constant $\frac{\epsilon}{\epsilon_0}$ placed in an initially uniform electric field, which at large distances from the sphere is directed along the z axis and has magnitude E_0. Both insdide and outside the sphere there are no charges. (a) Write down the potential equations for both inside and outside the sphere. (b) What are the boundary conditions for this case? (c) What is the electric field inside the sphere ?



4. (a) (8%) What are the Maxwell's equations in regions of space where there is neither charge nor current?

   (b) (7%) Derive the wave equations for $\vec{E}$ and $\vec{B}$ as the case in (a).

5. Show the following statements.

   (a) (7%) In the limit $kr \to \infty$, the expression for $\vec{A}(\vec{x})$ can be recast into

$$\lim_{kr\to\infty} \vec{A}(\vec{x}) = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\int \vec{J}(\vec{x}')e^{-ik(\vec{n}\cdot\vec{x}')}d^3\vec{x}' = \frac{\mu_0}{4\pi}\frac{e^{ikr}}{r}\sum_n \frac{(-ik)^n}{n!}\int \vec{J}(\vec{x}')(\vec{n}\cdot\vec{x}')^n d^3\vec{x}',$$

   where $\vec{n}$ is a unit vector in the direction of $\vec{x}$.

   (b) (7%) By applying the result in (a) to the electric dipole radiation field, the electric and magnetic fields take the form in the radiation zone

$$\vec{H} = \frac{ck^2}{4\pi}(\vec{n}\times\vec{p})\frac{e^{ikr}}{r},$$

$$\vec{E} = \sqrt{\frac{\mu_0}{\epsilon_0}}\vec{H}\times\vec{n},$$

   where $\vec{p} = \int \vec{x}'\rho(\vec{x}')d^3\vec{x}'$.

   (c) (6%) Calculate the radiation power per unit solid angle.

   Hint:

$$\frac{dP}{d\Omega} = \frac{1}{2}\mathrm{Re}[r^2\vec{n}\cdot\vec{E}\times\vec{H}^*].$$