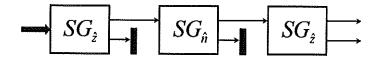
## DEPARTMENT OF PHYSICS, NATIONAL CHUNG CHENG UNIVERSITY QUALIFYING EXAM, FALL 2016 QUANTUM MECHANICS

- (1) A beam of spin-1/2 particles is sent through a sequence of three Stern-Gerlach (SG) analyzers (see the figure below), where the first and the third analyzers are aligned along the z-direction, while the second along a direction  $\hat{n}$  that makes an angle  $\theta$  in the xz-plane with respect to the z-axis. Note that the first two SG analyzers are designed to allow the spin-up component for each device orientation to pass.
  - (a) (10 points) What is the probability that particles transmitted through the first SG analyzer are measured to have spin down at the third SG analyzer?
  - (b) (10 points) How must the angle  $\theta$  of the second SG analyzer be oriented so as to maximize the probability that particles are measured to have spin down at the third SG analyzer? What is this maximum fraction?
  - (c) (5 points) What is the probability that particles have spin down at the third SG analyzer if the second SG analyzer is removed from the experiment? (*Hint*: The Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$



(2) (25 points) A particle of mass m moves in a one-dimensional potential

$$V(x) = -g\frac{\hbar^2}{m} \delta(x^2 - a^2),$$

where g is a positive dimensionless constant,  $\delta(x)$  the Dirac delta function, and a a positive constant. Determine the number of bound states for the particle within the potential according to the parameter g.

(3) Two non-interacting identical spin-1/2 particles of mass m are stored inside a one-dimensional harmonic potential

$$V(x) = \frac{1}{2}m\omega^2 x^2 ,$$

where  $\omega$  is a positive constant. The single-particle energy eigenfuctions for the one-dimensional simple harmonic potential are

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}} \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \left(y - \frac{d}{dy}\right)^n e^{-y^2/2} \Big|_{y = \sqrt{\frac{m\omega}{\hbar}}x}$$

with the corresponding eigenenergies

$$E_n(x) = \left(n + \frac{1}{2}\right)\hbar\omega, \qquad n = 0,1,2,3...$$

- (a) (10 points) Write down the energy and the total wavefunction(s) for the ground state of the two-particle system.
- (b) (15 points) Write down the energy and the total wavefunction(s) for the first excited state of the two-particle system.

(Hint: You must take into account the spin degrees of freedom and all possible degeneracies.)

(4) (25 points) A particle of mass m is initially at the ground state of the onedimensional infinite square-well potential

$$V(x) = \begin{cases} 0, & \text{if } 0 \le x \le a, \\ \infty, & \text{otherwise,} \end{cases}$$

where a is a positive constant. Suppose at time t = 0, an additional potential is switched on

$$\Delta V(x) = \begin{cases} V_0, & \text{if } 0 \le x \le a/2, \\ 0, & \text{otherwise,} \end{cases}$$

where  $V_0$  is a small positive constant (much less than the ground-state energy of the original infinite square-well). If the perturbing potential  $\Delta V(x)$  is switched off at time t = T, find the probability for the particle to be found in the first excited state of the original infinite square-well potential.